

# Optimal preventive maintenance policies for a two-component system with failure interaction

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In this paper we study a classical warranty policy 'free replacement policy' (FRW) for a two-component system with the following failure dependence between them: unit 1 causes random damages to unit 2 when it fails. While the failure of unit 2 is catastrophic and it destroys unit 1 and so the whole system. Imperfect maintenance by using virtual age method and system replacement are executed when the failure occurs to component 1 and the whole system respectively. Under both the non-renewable FRW and renewable FRW, from the manufacturers' point of view, the system expected total warranty costs are explored.

*Keywords:* warranty, maintenance, failure interaction, imperfect repair

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## 1 Introduction

In the study of multi-component system, the failure interaction [1] between the components can really affect the system reliability and the maintenance cost which should be taken into account. Murthy et al. [2] defined two types of dependency of 2-component-system. Nakagawa et al. proposed the so-called shock damage interaction between components which has been intensively studied in the literature.

In this paper, we consider a two-component system in parallel with shock damage dependence between them: each component 1 failure causes a random damage to unit 2 whereas component 2 failure causes the failure of component 1, resulting in the whole system failure. Component 1 follows a lifetime distribution while component 2 fails when its total damage exceeds a pre-determined level  $L$ . The damages caused to component 2 each time are identical and independently distributed.

## 2 Maintenance policy

Component 1 undergoes imperfect repair with virtual age method [3] and perfect corrective replacement is carried out when system failure occurs. To reduce the calculation complexity, the imperfect repair degree of component 1 is assumed to be a constant. Without regard to the system preventive maintenance, firstly the probability mass function of component 1 failure number by time  $t$  and its expectation by time  $t$  are derived respectively. Also, the system lifetime distribution function is obtained.

To avoid system failure, three system preventive maintenance policies [4] are considered:

- the system undergoes replacement at planned time  $T$ ;
- the system undergoes replacement at the  $N$ th failure of component 1;
- the system undergoes replacement at the planned time  $T$  or the  $N$ th failure of component 1 which occurs first.

It is noticed that under each preventive maintenance policy, the system is renewed after the maintenance. Thus the distribution function of the replacement intervals are identical and independent. The renewal theory is therefore adopted with respect to the calculation of the average costs in the infinite horizon. By considering only the

expectation of the first system failure time and the total maintenance cost on it, the long-run average costs under different maintenance policies are obtained.

Afterwards, the existence conditions which minimize the long run cost rates when the system is preventively repaired at time  $T$  or at the  $N$ th failure of component 1 are respectively determined. Particularly, when the system undergoes preventive replacement at planned time  $T$ , if component 1 has Weibull lifetime distribution and is minimally repaired at failure, it is showed that the optimal long-run average cost does not exist.

Furthermore, numerical and Monte Carlo simulations are presented to illustrate the method. Sensitivity analysis are discussed to elucidate the variation trend of cost rate under different system parameters.

## References

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