On the least squares estimation on stochastic fatigue crack growth model in martingale difference framework

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The motivation comes from a degradation mechanism that arises in fatigue crack growth (FCG). This study concerns the parameter estimation of a stochastic differential equation that describes the FCG phenomenon. The equation is written in the from of a stochastic regression model where the residuals are assumed martingale differences. In this framework, the parameter estimates are proved to be consistent.

The mechanism of crack evolution is modeled by a first order stochastic differential system, composed by a deterministic FCG equation which is perturbed by a stochastic process. The dynamical evolution of the increasing stochastic process Z_t, on $\mathbb{R}^d$, which stands for the crack length extension, is given by the following equation, Chiquet et al. [2009],

$$\frac{dZ_t}{dt} = \mathbf{C}(Z_t, X_t), \quad Z_0 = z_0, \tag{0.1}$$

where X_t is a process of space E. For the function $\mathbf{C}(\cdot,\cdot)$ we set the appropriate existence and unicity assumptions. In particular, we are concerned about the case that the rate function is expressed as the general stochastic equation of FCG, where a deterministic FCG equation of engineering origin, i.e., a function of Z_t say $m(Z_t)$, is multiplied by a stochastic process. In this case, the rate function is written as,

$$\mathbf{C}(Z_t, X_t) = m(Z_t)g(X_t), \tag{0.2}$$

where the perturbation process $g(X_t)$ is a deterministic (or fixed) function of X_t, $E \to \mathbb{R}_+$ and accounts for the randomness of the phenomenon. Due to its form in equation (0.2), the function $\mathbf{C}(\cdot,\cdot)$ admits the following representation, depending on the unknown parameter $\boldsymbol{\theta}_0$,

$$\log \mathbf{C}(z,x) = C_1(z, \boldsymbol{\theta}_0) + C_2(x), \text{ for } \boldsymbol{\theta}_0 \in \boldsymbol{\Theta} \subset \mathbb{R}^p, \ 0 < p < \infty. \tag{0.3}$$

The function $C_1(z, \boldsymbol{\theta}_0)$ is the parametric part of the model and $C_2(x) = \log g(X_t)$ are the respective residuals. Hence the observations are $V_n = \log \hat{\dot{Z}}_n$, where $\hat{\dot{Z}}_n$ stands for the estimated length variation on the instant t_n, i.e., $\hat{\dot{Z}}_n = \frac{Z_n - Z_{n-1}}{t_n - t_{n-1}}$, they are also written as

$$V_n = V(Z_{n-1}, X_n) = \log \mathbf{C}(Z_{n-1}, X_n) = C_1(Z_{n-1}, \boldsymbol{\theta}_0) + C_2(X_n). \tag{0.4}$$

The equation (0.4) is a stochastic regression model and, under the assumption that the residuals form martingale differences, see e.g. Jacob [2010], the conditional least squares estimates $\widehat{\boldsymbol{\theta}}_n$ of the parameter $\boldsymbol{\theta}_0$ are proved to be strongly consistent, i.e. $\widehat{\boldsymbol{\theta}}_n \to \boldsymbol{\theta}_0 \ a.s.$, Lai and Wei [1982].

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